NASA Technical Paper 1147

Study of the Use of a Nonlinear, Rate-Limited Filter on Pilot Control Signals

James J. Adams

APRIL 1978

NASA

NASA Technical Paper 1147

Study of the Use of a Nonlinear, Rate-Limited Filter on Pilot Control Signals

James J. Adams
Langley Research Center
Hampton, Virginia

NASA
National Aeronautics
and Space Administration

Scientific and Technical
Information Office

1978

SUMMARY

Analysis of pilot response in the performance of a closed-loop control situation has shown that a large remnant (noise) in the pilot's control output does not aid the control, but adds unwanted motion to the system response. The use of a filter on the pilot's control output could improve the performance of the pilot-aircraft system. What is needed is a filter with a sharp high frequency cutoff, no resonance peak, and a minimum lag at low frequencies. This investigation studies the usefulness of a nonlinear, rate-limited filter in performing the needed function. The effect of the nonlinear filter on the pilot's control output is compared with a linear first order filter and a no filter condition. An analytical study using pilot models and a simulation study using experienced test pilots were performed.

The results showed that the nonlinear filter does promote quick, steady maneuvering. The nonlinear filter attenuates the high frequency remnant and adds less phase lag to the low frequency signal than does the linear filter. In addition, the rate limit in the nonlinear filter can be set to be too restrictive, thereby causing an unstable pilot-aircraft system response.

INTRODUCTION

Analysis of pilot response in the performance of closed-loop control of dynamic systems has shown that the pilot's response is composed of a signal that is linearly related to the input signal and a random noise with a band pass equal to the band pass of the linear signal. The study which led to these conclusions is presented in reference 1 where the vehicle being controlled was an acceleration response plant K/s^2 and the pilot band pass was about 10 rad/sec. If the pilot's control output is considered to be composed of a signal linearly related to the input plus an uncorrelated noise, as is done in reference 1, then modeling the pilot as only the linear correlated part of his signal results in a lower error score than that obtained with the human pilot. Adding the noise results in an error score that matches the score obtained with the pilot. This study of reference 1 indicates that eliminating the noise from the pilot's control signal would improve his performance. In fly-by-wire control systems, it is possible to use a low pass filter on the pilot's control signal which, ideally, would eliminate the high frequency remnant signal without affecting the low frequency, linear part of the control signal. What is needed is a filter with a very sharp cutoff, but with no resonance peak, and with very little phase shift below the cutoff frequency. The purpose of the present investigation is to examine the usefulness of a nonlinear, rate-limited filter in providing this needed function. The nonlinear filter was compared with a no filter condition, and with a linear, first order filter. Higher order filters than first order were not considered for the linear filter.

Reference 2 is similar to the present study in many ways. In reference 2, flight tests were conducted with an elevator control booster which contained a variable rate limit. It was found that the rate limit could be restricted to 7°/sec with no detrimental effects on the controllability of the system. It should be noted that in reference 2 the control rate limit is not included in any stability augmentation loop closure, and the present study does not suggest that the filter be included in any stability augmentation loop. Reference 3 shows that including a rate limit in a stability augmentation loop can destroy the effectiveness of the stability augmentation. Including a rate limit in the pilot's control signal can also cause a deterioration in the control response of the pilot-aircraft system, but a much lower rate limit can probably be tolerated in the pilot's control signal than can be tolerated in a stability augmentation control system.

SYMBOLS

Values are given in SI Units. The measurements and calculations were made in U.S. Customary Units.

F_Z Z-axis force, N

g gravity, 9.81 m/sec^2

h altitude, m

I_Y moment of inertia, kg-m^2

K general gain

K_h pilot-model altitude-loop static gain, deg/m

K_n remnant static gain

K_θ pilot-model pitch-loop static gain, deg/deg

L_o lift for 1g

L_α $= -\dfrac{1}{mV}\dfrac{\partial F_Z}{\partial \alpha}$, per sec

M_q $= \dfrac{1}{I_Y}\dfrac{\partial M_Y}{\partial q}$, per sec

M_Y Y-axis moment, N-m

M_α $= \dfrac{1}{I_Y}\dfrac{\partial M_Y}{\partial \alpha}$, per sec^2

M_{δ_e}	$= \frac{1}{I_Y} \frac{\partial M_Y}{\partial \delta_e}$, per sec^2
m	mass, kg
q	pitching velocity, rad/sec or deg/rad
s	Laplace variable, per sec
t	time, sec
V	total velocity
V_{xo}	constant velocity in x-direction, m/sec
X,Y,Z	coordinate axes
α	angle of attack, rad
γ	flight-path angle, rad
δ_c	elevator command, rad or deg
δ_e	elevator deflection, rad or deg
$\delta_{e,p}$	pilot stick deflection, rad or deg
ζ_h	pilot model-aircraft system altitude mode damping ratio
ζ_{sp}	short period damping ratio
ζ_α	pilot model-aircraft system short period mode damping ratio
ζ_δ	pilot model-aircraft system control mode damping ratio
θ	pitch angle, rad or deg
λ	pilot model-aircraft system pitch mode root, rad/sec
Φ	frequency response phase angle, deg
ω	frequency, rad/sec
ω_h	pilot model-aircraft system altitude mode frequency, rad/sec
ω_{sp}	short period natural frequency, rad/sec
ω_α	pilot model-aircraft system short period mode frequency, rad/sec
ω_δ	pilot model-aircraft system control mode frequency, rad/sec

Subscripts:

c command

e error

A dot over a quantity denotes a derivative with respect to time.

EXPERIMENTAL PROCEDURE

The three filter configurations, no filter, nonlinear filter, and linear filter, were examined with pilot models in combination with a simple aircraft representation in an analytical study. These configurations were then examined with real pilots in combination with a complete aircraft representation in a fixed-base simulator. Three different tasks were executed in each case: (1) a step pitch attitude change, (2) a step altitude change, and (3) a sinusoidal altitude command. These tasks were performed with three different aircraft configurations which represented a medium speed condition at a Mach number of approximately 0.6 and an altitude of 7620 m, a high speed condition at a Mach number of approximately 1.0, and a low speed, low altitude condition.

The pilot model used in the analytical study was

$$\frac{\delta_{e,p}}{\theta_e} = \frac{K_\theta}{(1 + 0.2s)^2}$$

This equation relates pilot-control deflection to pitch-angle error. A block diagram of this model is shown as the inner pitch loop in figure 1. No lead term has been included in the pilot model because this study combines the pilot model with aircraft that have satisfactory handling qualities; it has been shown that no lead is required to represent a pilot's response with these aircraft. The lag time constant of 0.2 sec has been shown to be a proper value for the pilot model when aircraft with at least tolerable handling qualities are being controlled. The gain K_θ was adjusted to provide a pilot model-aircraft system response with the real root larger in magnitude than -0.4 rad/sec and a damping ratio of the oscillatory mode of motion greater than 0.1. These selections for the lag time constant and gain provide typical pilot-aircraft system characteristics. The selected pilot model was used without any further adjustment with each filter configuration to provide a clear indication of the effect of the filter on the system response. It was, of course, necessary to adjust the gain K_θ for each aircraft configuration to provide the desired system response, but the pilot-model coefficients were kept constant for each filter configuration. For altitude control the pilot model consisted of an outer loop added to the pitch control loop as shown in figure 1, with a constant gain K_h on the outer-loop control block. For the altitude control cases, the gains K_h and K_θ were adjusted to provide a system response with the lower frequency greater than about 1 rad/sec and the lowest damping ratio greater than 0.1. Again, these system characteristics are assumed to be typical for altitude control by a real pilot. With the high speed aircraft configuration, a small amount of lead was added to the pitch control loop pilot model in the altitude control system. Also, for all three aircraft configurations, a limit was placed on the pitch command (the output of the K_h block) in the altitude control systems. To complete the pilot models, a random white noise signal was filtered with a second order filter $K_n/(1 + 0.2s)^2$ and was added to the output of the pilot model to represent the remnant of the real pilot. The amplitude of this remnant signal was adjusted so that the variance of the remnant was between 40 and 50 percent of the total control signal. All these items have been shown to be reasonable for the representation of pilot response.

The pilot model was combined with a simplified, two-degree-of-freedom representation of the aircraft:

$$\dot{\alpha} - \dot{\theta} = -L_\alpha \alpha$$

$$\ddot{\theta} = M_q q + M_\alpha \alpha + M_{\delta_e} \delta_e$$

and the relationship for altitude

$$\dot{h} = V_{xo}(\theta - \alpha)$$

The coefficients for the three aircraft configurations are given in table I, together with the aircraft response characteristics. Also given are the pilot-model gains, K_θ and K_h, and the pilot model-aircraft system characteristics.

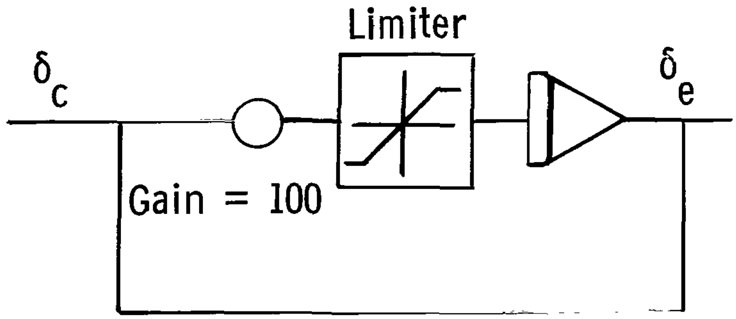
The nonlinear filter equations are:

$$\dot{\delta}_e = 100\delta_c - 100\delta_e$$

where $\dot{\delta}_e$ < Limit value, and

$$\dot{\delta}_e = \text{Limit value}$$

where $\dot{\delta}_e$ > Limit value. The rate limit was set as low as possible without causing unstable pilot control in order to insure the elimination of as much pilot remnant as possible. The value that was used was determined by a trial and error method. An analog diagram of the nonlinear filter is shown in sketch (a).



Sketch (a)

The linear filter was used with a frequency breakpoint of 5 rad/sec. This frequency value was chosen so that as much pilot remnant as possible would be rejected while as little low frequency lag as possible would be added to the system. The linear filter was mechanized in a straightforward manner.

In the simulation tests, three experienced test pilots performed the tasks. The simulator cockpit used by the pilots was equipped with a televised, out-the-window display of the horizon and a target airplane. The included angle of the display was 20° vertically and 35° horizontally. The control stick was a force stick with an unlimited, linear output and no hysteresis. The force stick was mounted on a rubber block base which gave it a small amount of rotational movement. The control sensitivity was adjusted so that it was satisfactory for all subjects.

The simulator was controlled by a five-degree-of-freedom aircraft representation with linear aerodynamics and nonlinear kinematics. These equations of motion are given in the appendix. The pilot performed the attitude control tasks with reference to the horizon and the altitude control tasks with reference to the target airplane. While the pilots were performing these longitudinal tasks, they also had to regulate the lateral directional response of the aircraft as an additional task. The target aircraft was driven so that it remained at a constant 183 m in front of the test aircraft. The target flew either straight and level or with a sinusoidal variation in altitude. The simulator equations of motion were solved with a digital computer that operated with a sample rate of 32 per sec. In order to represent the high frequency response of the nonlinear filter properly when it was operating on its linear region, a special local linearization computational technique, described in reference 4, was necessary.

RESULTS

Comparison of Filters

To illustrate the differences in the operation of the nonlinear and linear filters, the frequency response of the two devices can be compared. The data for the nonlinear filter can only be approximate and were obtained with an analog representation. Sinusoidal input signals with a number of different frequencies were applied to the nonlinear filter, and the time history of the output was recorded. The frequency response phase angle data were obtained by measuring the time difference in the zero crossing of the input and output and by using the formula

$$\Phi = (\Delta t)\omega\left(\frac{360}{2\pi}\right)$$

The amplitude ratio was obtained from the ratio of the peak values of the input and output. These data are shown in figures 2(a) and 2(b). In figure 2(a), data are shown which were computed with the ratio of the rate limit to the input amplitude set so that the breakpoint frequency would be approximately 5 rad/sec. These data are compared with the response of the linear filter. It can be seen that the phase lag for the nonlinear filter is less than the phase lag for the linear filter at low frequencies. This small phase lag at low frequencies for the nonlinear filter is the result of the 100-rad/sec breakpoint used in the linear part of the nonlinear filter. It can also be seen that the reduction in amplitude with increasing frequency is much steeper for the nonlinear filter than for the linear filter. The result is that the nonlinear filter would have less effect on low frequency signals; it would also attenuate high frequency signals better than would the linear filter.

Where the response of the linear filter is invariant with the amplitude of the input, the nonlinear filter response is affected by the amplitude of the input. This effect is illustrated in figure 2(b) where the approximate frequency response of the nonlinear filter for three different input amplitudes is presented. The figure shows that the larger the amplitude of the input, the more phase lag is created at any given frequency. This situation indicates a potential stability problem with large inputs for a system incorporating the nonlinear filter.

To indicate the effect on stability of the nonlinear filter for a typical pilot model-aircraft system, figure 3 is presented. The pilot model was simplified in this case by leaving out the remnant term. Figure 3(a) shows the response of a typical system with the nonlinear filter included but with the rate limit set so high that it does not come into effect. The commanded pitch-angle change is 5° in this case. Figure 3(b) shows the response of the same system with the nonlinear filter rate limit set so that it does come into operation. In comparison with figure 3(a), figure 3(b) shows that the nonlinear filter does noticeably change the control moment time history, but there is no noticeable effect on the pitch-angle time history. Figure 3(c) shows the response of the same system used in figure 3(b) to a 10° pitch-angle change command. In this last case the initial overshoot in pitch angle is noticeably larger in proportion to the steady-state value of pitch angle than in the case with the 5° pitch-angle change. This change in the stability of the response of the system with an increase in the size of the input command illustrates a possible disadvantage of the nonlinear filter.

In this report, control action is presented as the normalized control moment $M_{\delta_e}\delta_e$; control deflection is not used. The purpose for this particular method of data presentation is to generalize the results rather than to leave them as the function of a particular control effectiveness value.

Pilot Model Analysis

To test the usefulness of the nonlinear filter, a study using pilot models for both attitude and altitude control was undertaken. This study compared the nonlinear filter with both no filter and a linear, first order filter. The comparison was made with each of three different aircraft configurations. The nonlinear filter rate limits were established initially in these tests by noting the maximum rate required in the 5° attitude change maneuver and by setting the rate limit at one-half this maximum value. Further restrictions in the rate limit were then tried.

The first aircraft configuration to be discussed represents a fighter aircraft at medium speed. The aircraft speed was 214 m/sec, and the aircraft short period response characteristics were $\omega_{sp}^2 = 20\ \text{rad}^2/\text{sec}^2$, $2\zeta_{sp}\omega_{sp} = 5$ rad/sec ($\omega_{sp} = 4.48$ rad/sec, $\zeta_{sp} = 0.56$). The results obtained for a step change in pitch angle when the filters were inserted in a system containing this aircraft and the typical pilot model are shown in figure 4. The figure shows a reduction in the pitching motion activity as illustrated by the amplitude of pitching velocity q. This reduced pitching activity is evident when compared with the no filter condition. The nonlinear filter brings about a greater reduction in pitching activity than does the linear filter. Each filter reduces the effect of the pilot remnant, but the linear filter also reduces the damping of the osillatory mode of motion of the system. This reduction in system damping is illustrated more clearly in figure 5 where the pilot remnant has been removed from the pilot model.

Similar results were obtained when a step change in altitude was computed using the multiloop pilot model. These results are shown in figure 6, and again a decrease in system damping occurs when the linear filter is added to the pilot model-aircraft system; slightly less pitching motion activity occurs with the nonlinear filter than with the linear filter. However, in this case the system with no filter demonstrated the smallest pitching activity.

In the consideration of the high speed aircraft configuration ($V = 305$ m/sec, $\omega_{sp}^2 = 100\ \text{rad/sec}^2$, $2\zeta_{sp}\omega_{sp} = 3$ rad/sec), a clear reduction in pitching activity occurred with the nonlinear filter in comparison with the no filter configuration or the linear filter. The results are shown in figure 7, where the response to a step change in altitude is shown. The same result was obtained with the step change in pitch-angle computation. It should be mentioned again that a small amount of pilot lead (a lead time constant of 0.2 sec) was used in the computation of the altitude change shown in figure 7. This amount of lead is an addition that a pilot, attempting to improve the system response, is likely to try in his control response.

With the low speed aircraft configurations ($V = 122$ m/sec, $\omega_{sp}^2 = 5$ rad/sec^2, $2\zeta_{sp}\omega_{sp} = 5$ rad/sec), the results of the step pitch angle and step altitude change were the same as for the first two configurations. One test in which the filters had a pronounced effect with the low speed aircraft configuration was in following a sinusoidal altitude command. A typical computed run is shown in figure 8. The command in this case was a sine wave with a period of 30 sec and an amplitude of 120 m. In these cases it was necessary to remove the limit in the pilot model on pitch-angle command because an angle larger than the limit was required to perform the maneuver. As a result, the system instability that results without the pitch-angle command limit can be seen to occur at the end of the run.

A summary of the results obtained from these computations is presented in table II which contains the root-mean-square values for altitude error for all three aircraft configurations with all three filters. To show at a glance the effect of the nonlinear filter and the linear filter, these root-mean-square error values have been normalized (to 100) to the no filter case. The results show that with the low speed aircraft configuration, both the nonlinear filter and the linear filter provide considerable improvement in the sinusoidal command-following ability of the pilot model-aircraft system. Less improvement was provided by the two filters with the high speed aircraft, and with the medium speed aircraft, the filters reduced the accuracy of the sinusoidal altitude following.

Simulation Tests

Three experienced test pilots (P, K, and E) served as subjects in the simulation tests. Each pilot tested each filter configuration with each of the three aircraft configurations. In this investigation the rate limit in the nonlinear filter was set by combining the pilot model with the five-degree-of-freedom, nonlinear aircraft representation and by noting the maximum control moment rate that was required in a 3° pitch-angle change maneuver. The rate limit was then set at one-half of this maximum value. Although this method of setting the rate limit proved to be very useful in determining the initial value, preliminary tests showed that the rate limit could be restricted a little more. The task of following the target airplane, which was moving vertically with a sinusoidal variation with a period of 30 sec and an amplitude of 120 m, proved to be the most sensitive test of required control moment rate; therefore, this task was used in this preliminary investigation. The author was the subject in these tests (subject G). Sample tests with the high speed aircraft are shown in figure 9. With the initial limit value for $M_{\delta_e}\dot{\delta}_e$ of $450^\circ/\text{sec}^3$, the pilot performed the maneuver with no difficulty. When the limit value was reduced to $250^\circ/\text{sec}^3$, the pilot experienced some difficulty at first, as is indicated by the one cycle of a divergent oscillation that can be seen between the 5- and 25-sec marks in figure 9. However, the pilot made the necessary adjustment and regained control. It was concluded from this test that the limit value of $250^\circ/\text{sec}^3$ was close to the lowest useful value for the rate limit, and this value was used in the remainder of the investigation with the high speed aircraft configuration. Similar tests were made with the other two aircraft configurations; consequently, values of $M_{\delta_e}\dot{\delta}_e$ of

$80^{\circ}/sec^3$ for the medium speed configuration and $60^{\circ}/sec^3$ for the low speed configuration were selected for the remainder of the investigation.

The results for the pilots in the simulator closely parallel those obtained with the pilot model in the analytical study. Time history records obtained with pilot P for the step change in pitch angle are shown in figure 10. These results are typical for all the subjects. The pilots conducted these tests in a systematic manner by first performing a very slow maneuver (using a low gain) which they were sure would be well damped. They increased the maneuver rate in the next try, and then made a final maneuver which was done as rapidly as they believed they would ever perform the maneuver. This final maneuver should compare with the pilot model results. Figure 10 shows that in comparison with the response with either the no filter or nonlinear filter, there is a reduction in system damping for the rapid maneuver with the linear filter included in the system. In addition, the pitching activity is lowest with the nonlinear filter. Figure 11 shows the step altitude change maneuver, and again, the pitching activity is slightly less with the nonlinear filter than with either the no filter or the linear filter configurations.

With the high speed aircraft configuration, the pitching activity is clearly the smallest with the nonlinear filter in both the attitude change and the altitude change tasks. These results are shown in figures 12 and 13 where pilot P was the subject. These figures show that not only does the nonlinear filter reduce the random noise generated by the pilot, but also it does not effect the linear portion of the pilot's response. Therefore, with the nonlinear filter in the system, the final steady-state condition of the maneuver is arrived at quickly and with only a small oscillation about this steady-state value.

With the low speed aircraft configuration, the most pronounced effect in the simulation tests occurred, as it did in the pilot-model analysis, in the task of following a sinusoidal altitude command. A set of typical time histories is shown in figure 14. There was a great deal of learning involved in this task. The performance measure used was the difference in the altitude of the target airplane and that of the controlled airplane, but the pilots tended to want only to keep the gun sight pipper on the target. As they learned that a good score resulted from staying at the same altitude as the target and, at the same time, learned to use a small amount of lead to accomplish this task, the scores improved by a large amount. To show the final result, the last three scores of the one subject who performed a complete set of tests are given in table II. It can be seen that no improvement in root-mean-square values of the altitude error was provided by either the nonlinear filter or the linear filter with the medium speed aircraft configuration, there was some improvement with the high speed aircraft, and there was a noticeable improvement due to the filters with the low speed aircraft. These results closely match the results obtained in the pilot model analysis.

The pilots were asked to rank the different filter configurations as best (1), in between (2), and worst (3). These rating data are given in table III. It can be seen that the pilots did not agree in their rankings. Further, individual pilots varied in their ratings when different aircraft configurations were involved. The effect of the filter, as shown in the time histories presented previously, was small; this probably prevented the pilots from reaching an agreement in rankings given the small amount of experience they had with the different filters during the present experiment. Nevertheless, based on the improved performance shown in the time histories, the nonlinear filter obviously provides real benefits; it deserves further consideration.

In the present investigation, the filters were not located inside any stability augmentation loops. The filters were meant to be inside pilot-loop closures only, and, therefore, the aircraft was represented as having no stability augmentation. Even had the aircraft included stability augmentation, the intention was to locate the filter outside these loops.

The nonlinear filter was also tried on the aileron control system. Tests were made both with pilot models in an analytical study and with real pilots in the simulation study. In each situation it was found that a small amount of rate restriction caused a very noticeable deterioration in the stability of the system response. For this reason the use of the nonlinear filter, as defined in this study, is recommended for use only in the elevator control system.

In the present investigation, the rate limit in the nonlinear filter was set individually for each aircraft configuration and was a different value for each aircraft. This situation indicates that if the nonlinear filter were to be used in an aircraft with a large flight envelope, the rate limit value would have to be scheduled as a function of flight conditions to achieve the best results possible. This scheduling problem was by-passed in the present investigation.

During the present study there were no instances found where the nonlinear filter caused a divergent oscillation in an attitude control task. However, there were cases involving altitude control in which pilot-induced unstable oscillations did occur. In the pilot-model analytical study, it was found that a rate restriction which was satisfactory for the attitude control task did cause an unstable oscillation in the altitude change task if the pitch-angle command limit was not included in the pilot model. Including the pitch-angle command limit eliminated the problem. In the simulation study it was found that using a rate limit in the nonlinear filter with a limit value smaller than the value reported in this study caused pilot-induced unstable oscillations in altitude control tasks. Figure 9(b) is an example of a borderline case. A rate restriction that is too great must clearly be avoided. The critical tasks where trouble might arise are tasks that require rapid and accurate altitude regulation. Formation flying, short range air-to-air combat, and landing are examples of such tasks. Large altitude changes such as those required in navigation tasks, but which do not require rapid and accurate response, would probably not be critical.

CONCLUSIONS

Analytical studies using pilot models and simulation studies using pilot subjects have led to the following conclusions about the usefulness of a nonlinear, rate-limited, pilot pitch control filter:

1. The pilot-model analytical study showed that the nonlinear filter did eliminate enough of the pilot's remnant (noise) so that smoother maneuvering, with smaller amplitude in the pitching velocity peak values, could be obtained. The linear filter not only rejected the pilot's remnant, but also reduced the damping of the basic pilot-aircraft system modes of motion.

2. Time histories obtained with experienced research pilots confirmed the results of the analytical study because the nonlinear filter did promote rapid, smooth maneuvers. However, pilot rankings did not confirm the results shown in the time histories. This lack of confirmation from pilot rankings is to be expected because of the small effect of the filter and the lack of extensive experience afforded by these abbreviated tests.

3. The study showed that instabilities in the pilot-aircraft system can result when the value of the rate limit in the nonlinear filter is set too low. The study shows the advantages that can be obtained with the nonlinear filter; further development may alleviate the instability problem.

Langley Research Center
National Aeronautics and Space Administration
Hampton, VA 23665
February 24, 1978

APPENDIX

EQUATIONS OF MOTION FOR THE SIMULATION STUDY

The equations of motion used for the pilot simulation experiment were:

$$A_x = 0$$

$$A_y = V_{xo} Y_\beta \beta$$

$$A_z = -V_{xo}(L_\alpha \alpha - L_o)$$

$$\dot{p} = L'_p p + L'_\beta \beta + L'_r r + L'_{\delta_a} \delta_a$$

$$\dot{q} = M_\alpha \alpha + M_q q + M_{\delta_e} \delta_e$$

$$\dot{r} = N'_r r + N'_\beta \beta + N'_p p + N'_{\delta_r} \delta_r$$

$$\dot{\phi} = p + q \sin\phi \tan\theta + r \cos\phi \tan\theta$$

$$\dot{\theta} = q \cos\phi - r \sin\phi$$

$$\dot{\psi} = \frac{r \cos\phi + q \sin\phi}{\cos\theta}$$

$$l_1 = \cos\psi \cos\theta$$

$$m_1 = \cos\psi \sin\theta \sin\phi - \sin\psi \cos\phi$$

$$n_1 = \cos\psi \sin\theta \cos\phi + \sin\psi \sin\phi$$

$$l_2 = \sin\psi \cos\theta$$

$$m_2 = \sin\psi \sin\theta \sin\phi + \cos\psi \cos\phi$$

$$n_2 = \sin\psi \sin\theta \cos\phi - \cos\psi \sin\phi$$

$$l_3 = -\sin\theta$$

$$m_3 = \cos\theta \sin\phi$$

$$n_3 = \cos\theta \cos\phi$$

$$\dot{V}_x = l_1 A_x + m_1 A_y + n_1 A_z$$

$$\dot{V}_y = l_2 A_x + m_2 A_y + n_2 A_z$$

$$\dot{V}_z = l_3 A_x + m_3 A_y + n_3 A_z + g$$

$$u = l_1 V_x + l_2 V_y + l_3 V_z$$

$$v = m_1 V_x + m_2 V_y + m_3 V_z$$

$$w = n_1 V_x + n_2 V_y + n_3 V_z$$

$$V = \left(V_x^2 + V_y^2 + V_z^2\right)^{1/2}$$

$$\alpha = \tan^{-1} \frac{w}{u}$$

$$\beta = \sin^{-1} \frac{v}{V}$$

where

A_X, A_Y, A_Z body-axis components of acceleration, m/sec^2

F_Y side force, N

I_X, I_Z moments of inertia, kg-m^2

I_{XZ} product of inertia, kg-m^2

L_o $= \dfrac{g}{V_{x_o}}$, per sec

L_p $= \dfrac{1}{I_X}\dfrac{\partial M_X}{\partial p}$, per sec

L_r $= \dfrac{1}{I_X}\dfrac{\partial M_X}{\partial r}$, per sec

L_β $= \dfrac{1}{I_X}\dfrac{\partial M_X}{\partial \beta}$, per sec^2

L_{δ_a} $= \dfrac{1}{I_X}\dfrac{\partial M_X}{\partial \delta_a}$, per sec^2

L_i' $= \left(1 - \dfrac{I_{XZ}^2}{I_X I_Z}\right)^{-1}\left(L_i + \dfrac{I_{XZ}}{I_X} N_i\right)$ $(i = \beta,\ p,\ r,\ \delta_a,\ \text{and}\ \delta_r)$

L_p' = -2.74, per sec

L_r' = 2.058, per sec

L_β' = -42.14, per sec^2

L_{δ_a}' = -10.0, per sec^2

M_X rolling moment, N-m

M_Z yawing moment, N-m

N_p $= \dfrac{1}{I_Z}\dfrac{\partial M_Z}{\partial p}$, per sec

$N_r \quad = \dfrac{1}{I_Z}\dfrac{\partial M_Z}{\partial r}$, per sec

$N_\beta \quad = \dfrac{1}{I_Z}\dfrac{\partial M_Z}{\partial \beta}$, per sec^2

$N_{\delta_a} \quad = \dfrac{1}{I_Z}\dfrac{\partial M_Z}{\partial \delta_a}$, per sec^2

$N_i' \quad = \left(1 - \dfrac{I_{XZ}^2}{I_X I_Z}\right)^{-1}\left(N_i + \dfrac{I_{XZ}}{I_Z} L_i\right)$ $\quad (i = \beta,\ p,\ r,\ \delta_r,$ and $\delta_a)$

N_p' = 0.0148, per sec

N_r' = -0.278, per sec

N_β' = 5.54, per sec^2

N_{δ_r}' = -10.0, per sec^2

p,r rolling and yawing velocity, rad/sec

u,v,w body-axis components of velocity, m/sec

V_x, V_y, V_z vertical-axis components of velocity, m/sec

$Y_p \quad = \dfrac{1}{mV}\dfrac{\partial F_Y}{\partial p}$, per sec

$Y_r \quad = \dfrac{1}{mV}\dfrac{\partial F_Y}{\partial r}$, per sec^2

$Y_\beta \quad = \dfrac{1}{mV}\dfrac{\partial F_Y}{\partial \beta}$, per sec

Y_β = -0.159, per sec

β sideslip angle, rad

δ_a, δ_r aileron and rudder deflection, rad or deg

ψ, ϕ yaw and roll angle, rad

REFERENCES

1. Adams, James J.; and Bergeron, Hugh P.: A Synthesis of Human Response in Closed-Loop Tracking Tasks. NASA TN D-4842, 1968.

2. Mathews, Charles W.; Talmage, Donald B.; and Whitten, James B.: Effects on Longitudinal Stability and Control Characteristics of a Boeing B-29 Airplane of Variations in Stick-Force and Control-Rate Characteristics Obtained Through Use of a Booster in the Elevator-Control System. NACA Rep. 1076, 1952. (Supersedes NACA TN 2238.)

3. Schmidt, Stanley F.; and Triplett, William C.: Use of Nonlinearities To Compensate for the Effects of a Rate-Limited Servo on the Response of an Automatically Controlled Aircraft. NACA TN 3387, 1955.

4. Barker, Lawrence E., Jr.; Bowles, Roland L.; and Williams, Louise H.: Development and Application of a Local Linearization Algorithm for the Integration of Quaternion Rate Equations in Real-Time Flight Simulation Problems. NASA TN D-7347, 1973.

TABLE I.- AIRCRAFT STABILITY DERIVATIVES AND OPEN- AND CLOSED-LOOP CHARACTERISTICS

Parameter	Medium speed aircraft	High speed aircraft	Low speed aircraft
Aircraft configuration			
V_{xo}, m/sec	214	305	122
L_α	1.3	1.3	.6
L_o	.0461	.0322	.0805
M_α	-15.2	-97.8	-2.36
M_q	-3.70	-1.70	-4.40
M_{δ_e}	-10.0	-10.0	-10.0
ω_{sp}^2, rad/sec	20	100	5
$\zeta_{sp}\omega_{sp}$, rad/sec	5	3	5
ω_{sp}, rad/sec	4.48	10.00	2.24
ζ_{sp}	.55	.15	1.11
Pilot model-aircraft system attitude control characteristics			
K_θ, deg/deg	2.4	6.0	1.5
λ, rad/sec	-.890	-.567	-.526
ω_α, rad/sec	3.92	9.30	2.75
ζ_α	.10	.11	.10
ω_δ, rad/sec	7.55	6.25	7.25
ζ_δ	.89	.83	.96
Pilot model-aircraft system altitude control characteristics			
K_θ, deg/deg	2.4	6.0	1.5
K_h, deg/m	1.75	1.23	1.54
ω_h, rad/sec	1.35	1.02	.740
ζ_h	.125	.103	.244
ω_α, rad/sec	3.85	9.33	2.67
ζ_α	.156	.224	.133
ω_δ, rad/sec	7.55	6.47	7.29
ζ_δ	.89	.82	.95

TABLE II.- ROOT-MEAN-SQUARE ALTITUDE ERROR IN

SINUSOIDAL ALTITUDE COMMAND TASK

(a) Pilot-model results

Aircraft configuration	Normalized error for -		
	Filter configuration		
	No filter	Nonlinear filter	Linear filter
Medium speed	100	105.5	105.5
High speed	100	94	93
Low speed	100	90.5	90

(b) Piloted results; subject G

Trial	Error in meters for -		
	No filter	Nonlinear filter	Linear filter
	Medium speed aircraft		
1	12.2	12.8	11.5
2	9.9	6.9	8.5
3	6.2	7.6	6.8
	High speed aircraft		
1	9.7	8.7	8.8
2	9.7	5.8	8.4
3	8.2	6.6	8.4
	Low speed aircraft		
1	30.0	20.8	25.0
2	24.6	20.2	21.1
3	21.7	21.0	27.7

TABLE III.- PILOT RANKING OF FILTER CONFIGURATIONS

<table>
<tr><th rowspan="3">Pilot</th><th colspan="3">Pilot ranking for -</th></tr>
<tr><th colspan="3">Filter configuration</th></tr>
<tr><th>No filter</th><th>Nonlinear filter</th><th>Linear filter</th></tr>
<tr><td colspan="4">Medium speed aircraft</td></tr>
<tr><td>P</td><td>3</td><td>1</td><td>2</td></tr>
<tr><td>K</td><td>2</td><td>3</td><td>1</td></tr>
<tr><td>E</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td colspan="4">High speed aircraft</td></tr>
<tr><td>P</td><td>2</td><td>2</td><td>1</td></tr>
<tr><td>K</td><td>1</td><td>3</td><td>2</td></tr>
<tr><td>E</td><td>3</td><td>1</td><td>2</td></tr>
<tr><td colspan="4">Low speed aircraft</td></tr>
<tr><td colspan="4">All filters were ranked as equal by all pilots.</td></tr>
</table>

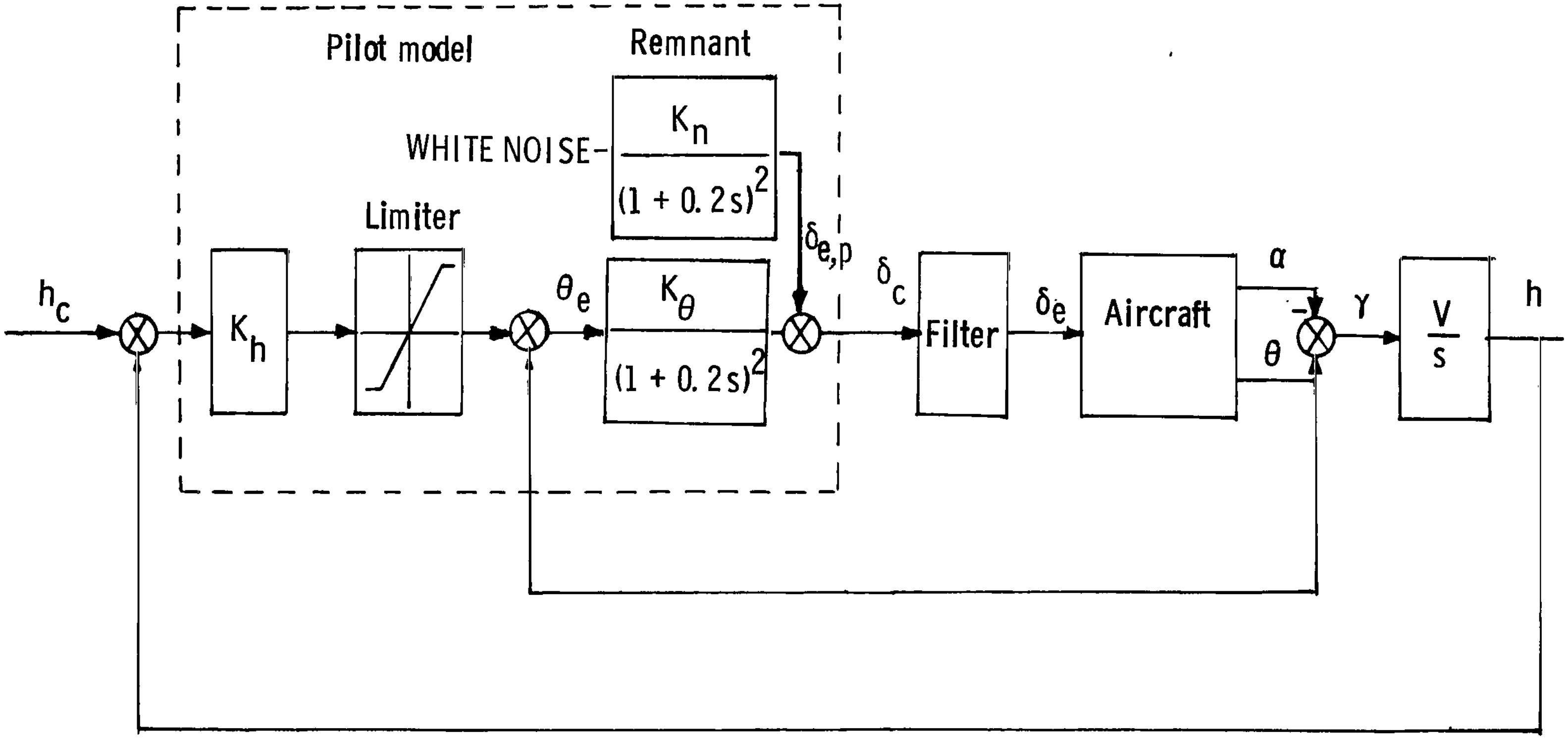
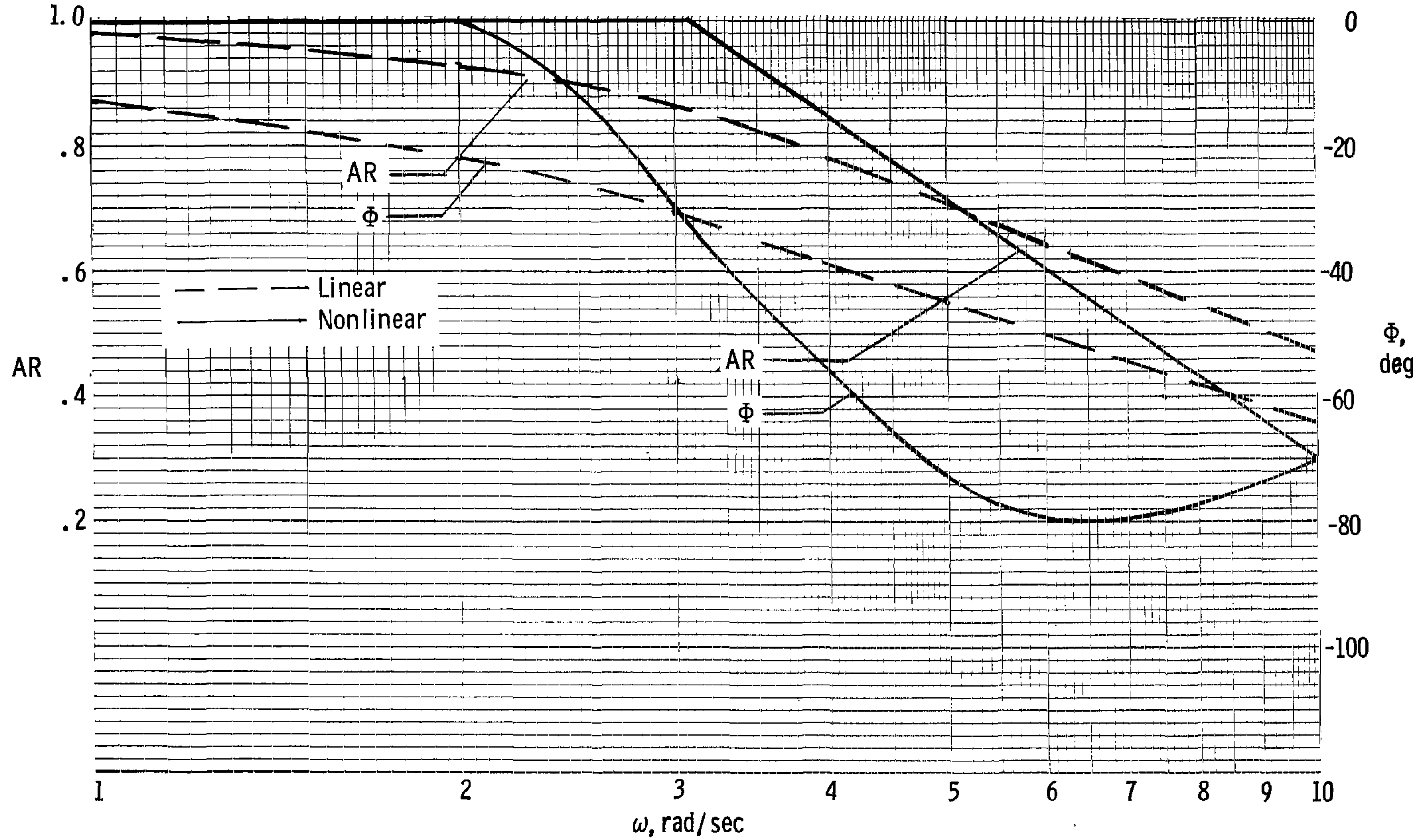
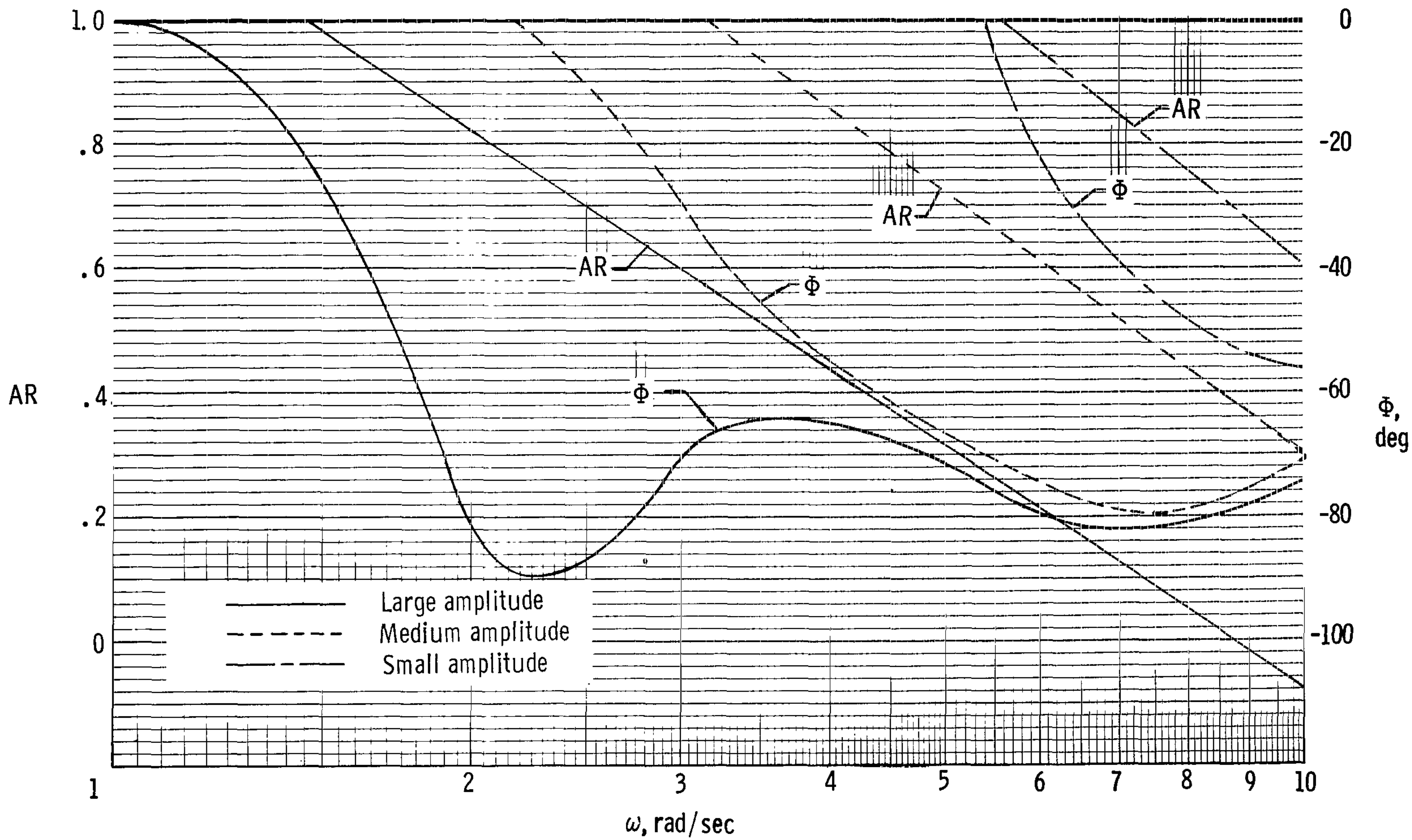


Figure 1.- Block diagram of pilot model-aircraft system.



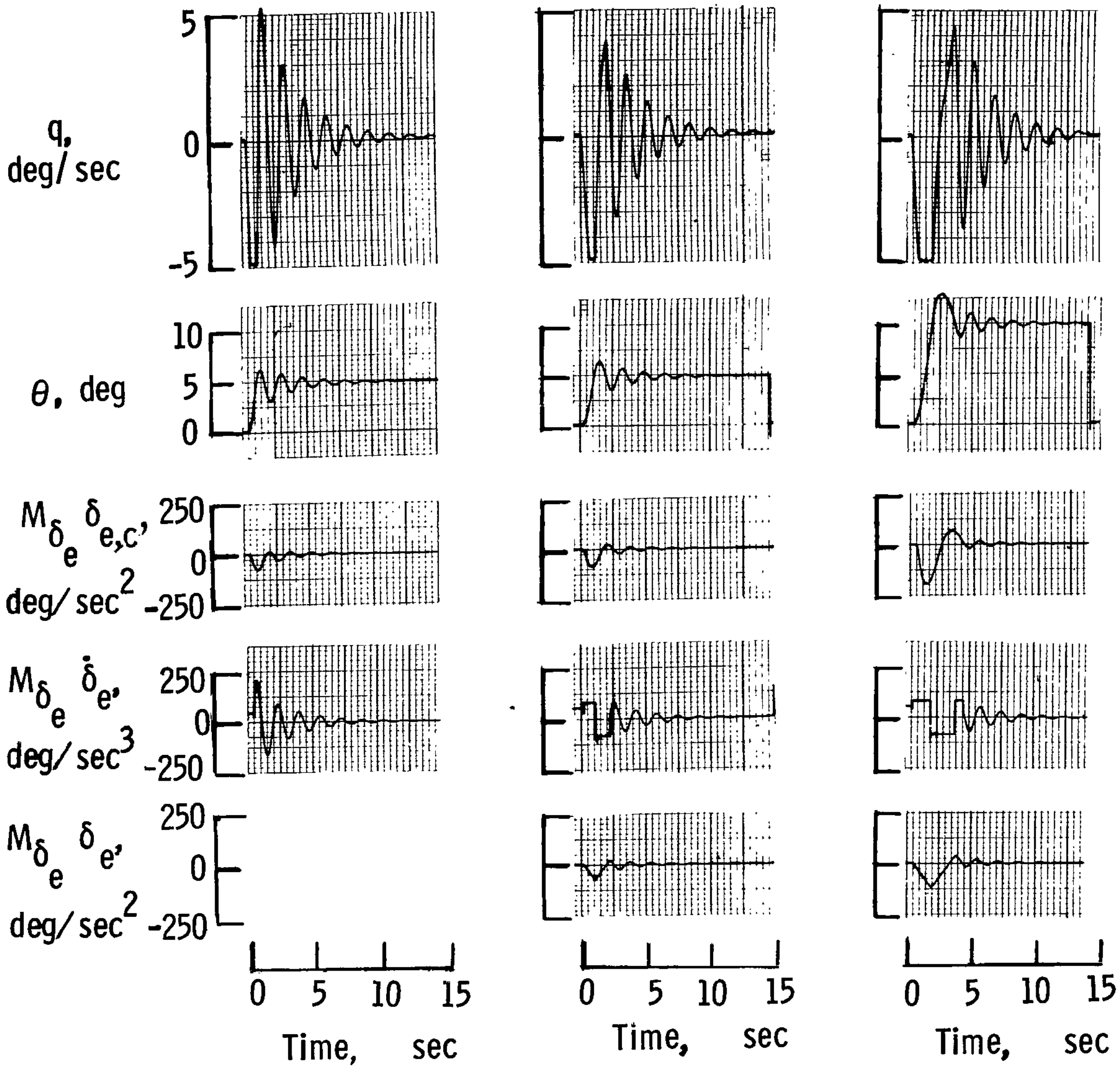
(a) Comparison of nonlinear and linear filter.

Figure 2.- Filter frequency response. (AR denotes amplitude ratio.)



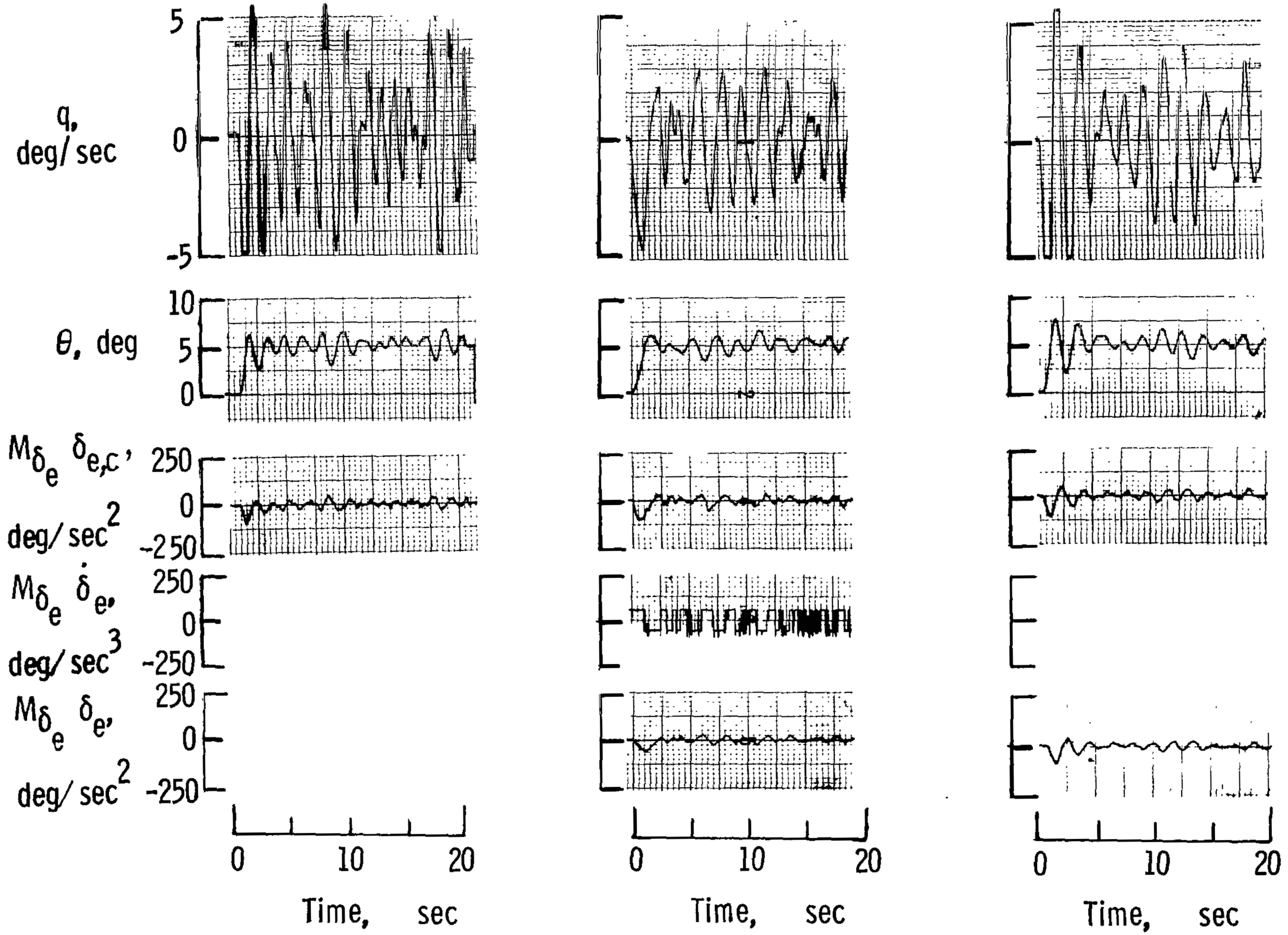
(b) Nonlinear filter with three different input amplitudes.

Figure 2.- Concluded.



(a) No rate limit. (b) Rate limit in effect. (c) Large command.

Figure 3.- Effect of nonlinear rate limit on system stability; medium speed aircraft.

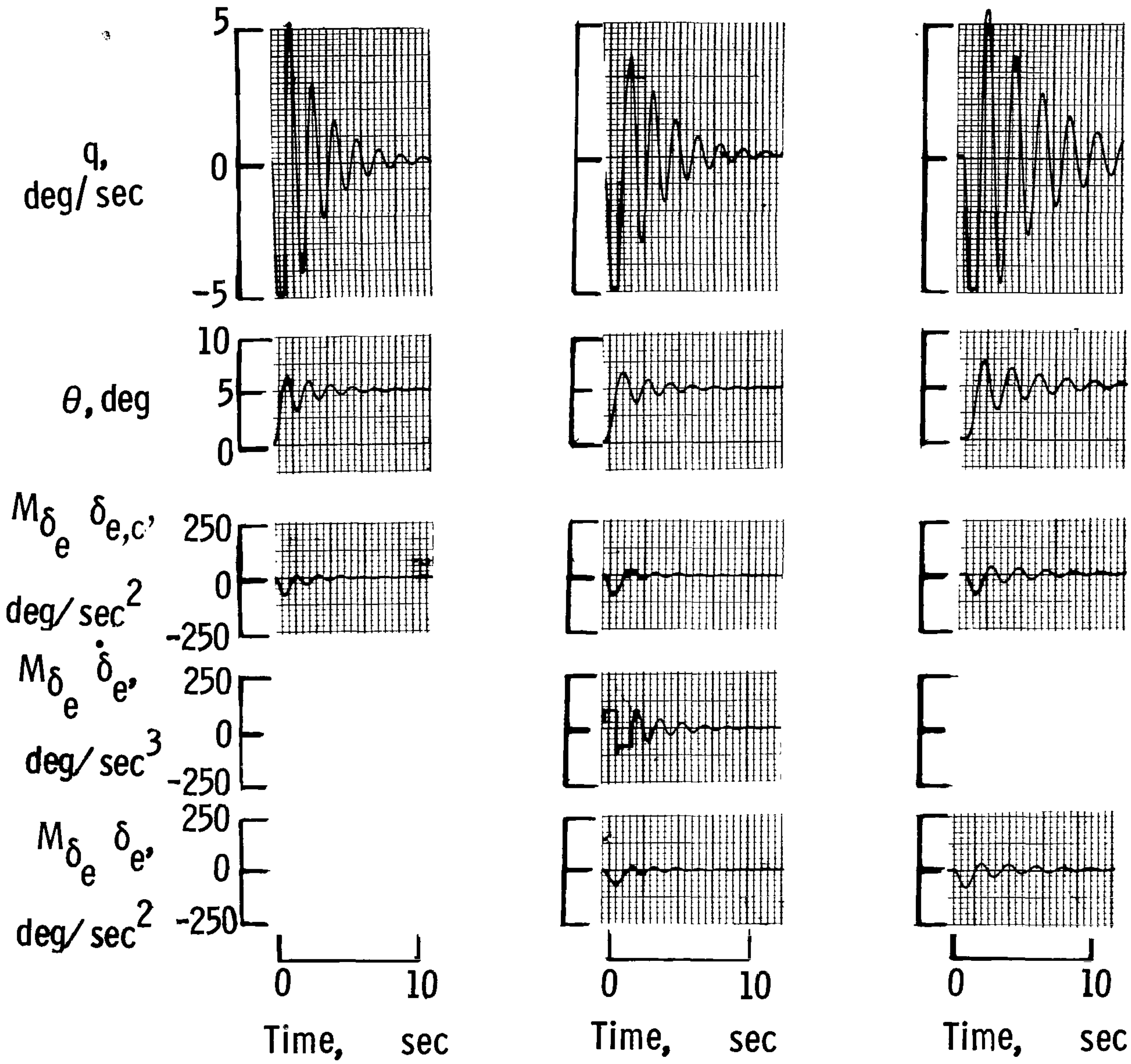


(a) No filter.

(b) Nonlinear filter.

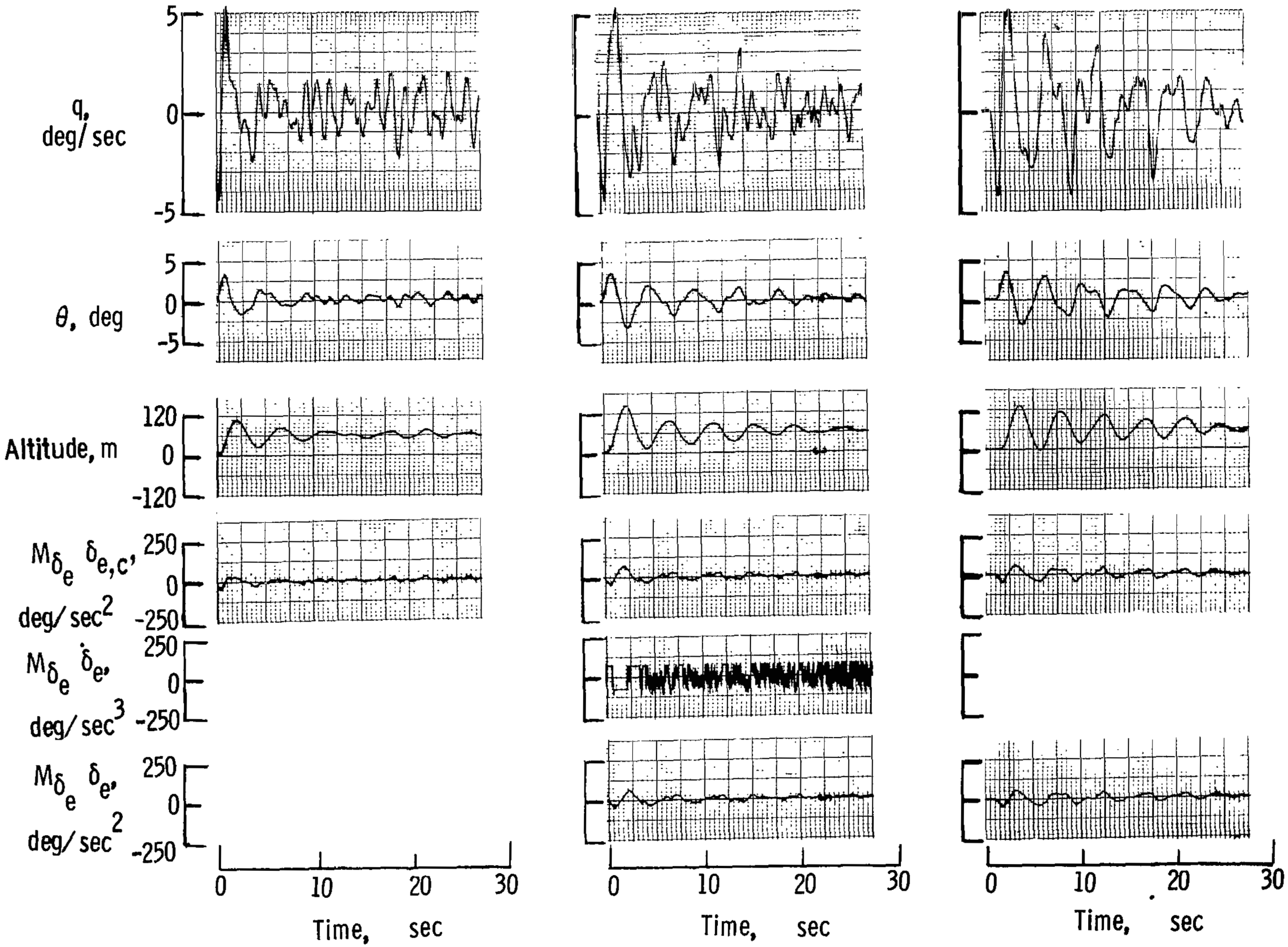
(c) Linear filter.

Figure 4.- Step pitch-angle change; medium speed aircraft.



(a) No filter. (b) Nonlinear filter. (c) Linear filter.

Figure 5.- Step pitch-angle change; medium speed aircraft; remnant omitted from pilot model.

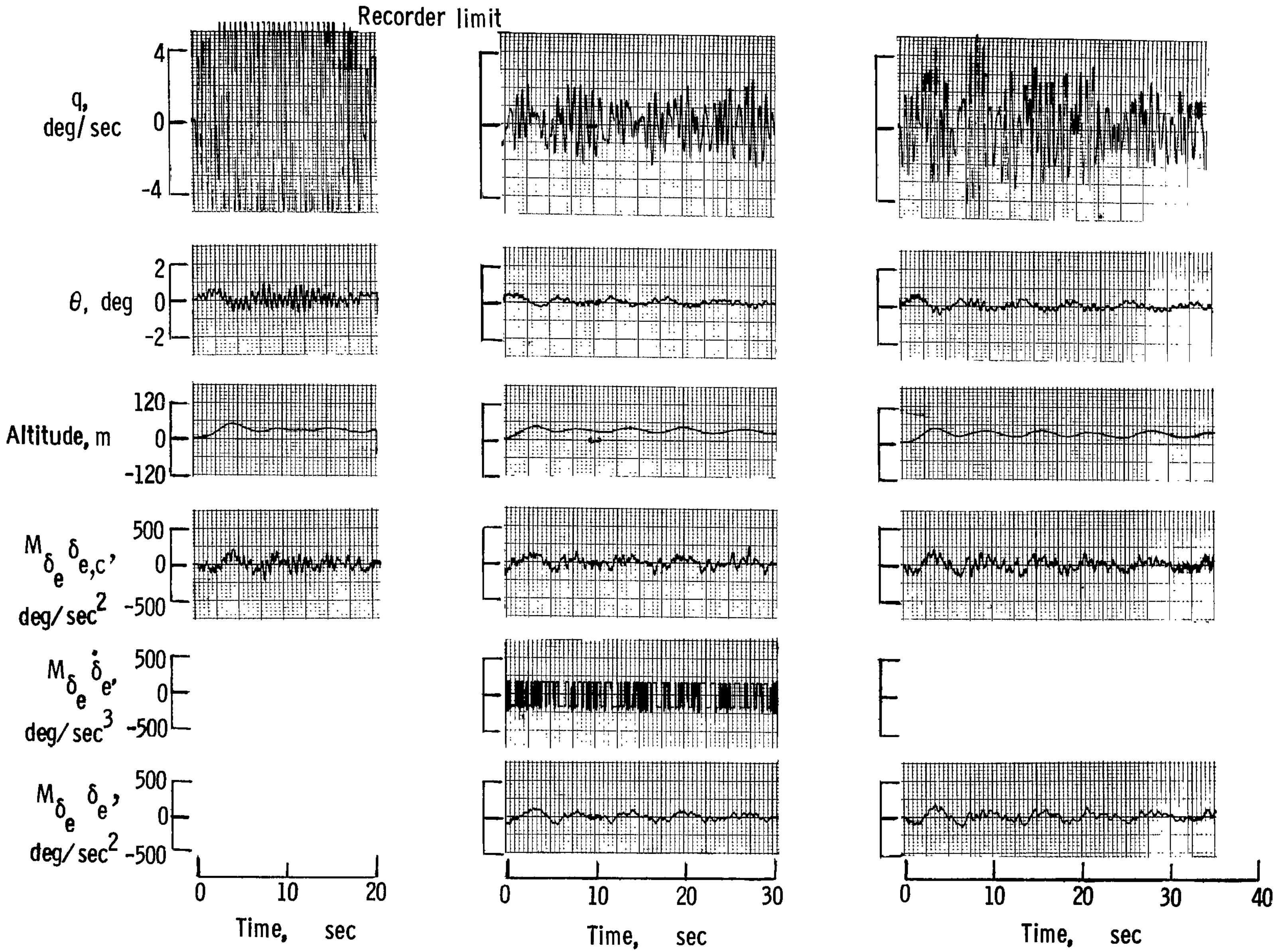


(a) No filter.

(b) Nonlinear filter.

(c) Linear filter.

Figure 6.- Step altitude change; medium speed aircraft.



(a) No filter.

(b) Nonlinear filter.

(c) Linear filter.

Figure 7.- Step altitude change; high speed aircraft.

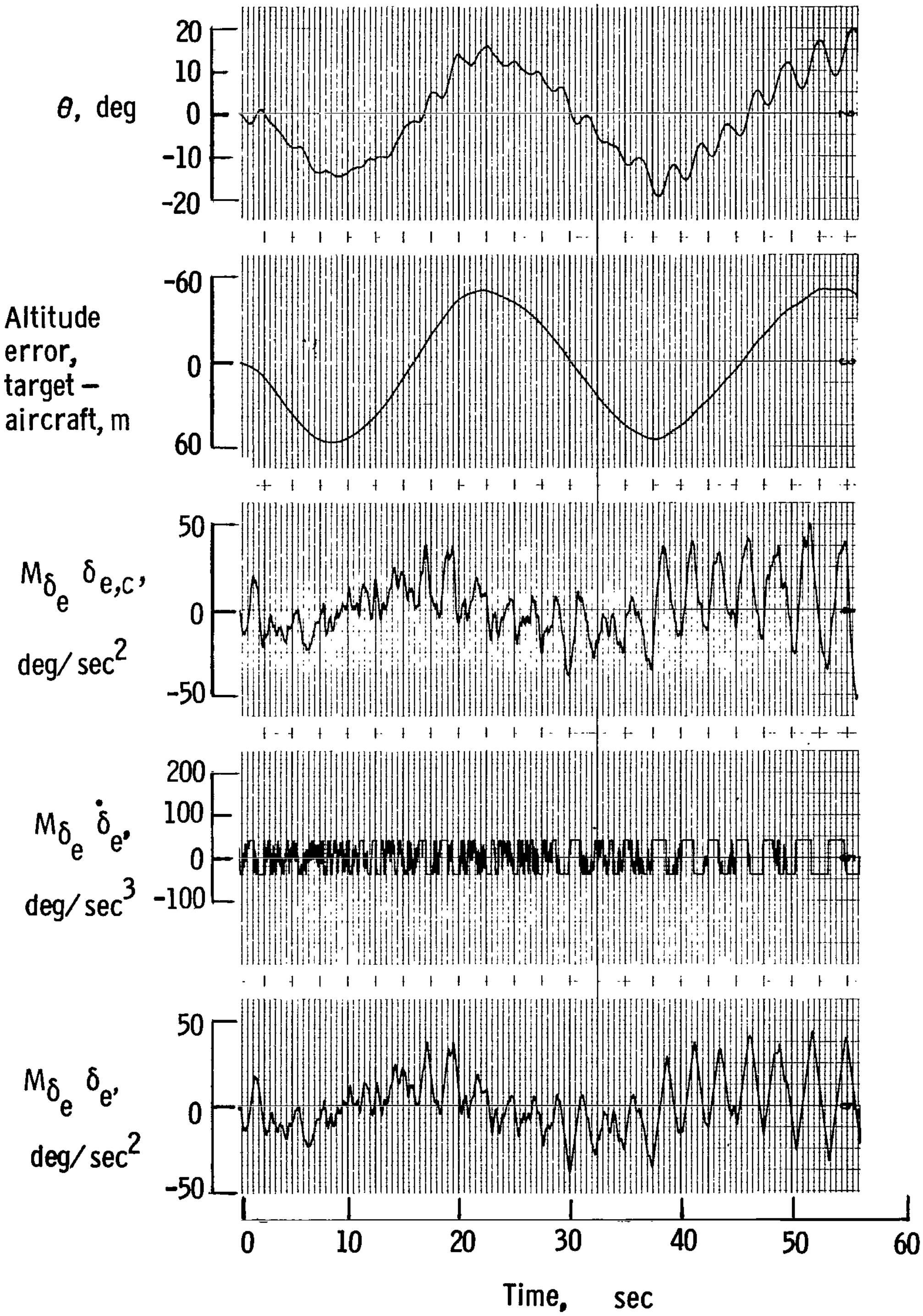
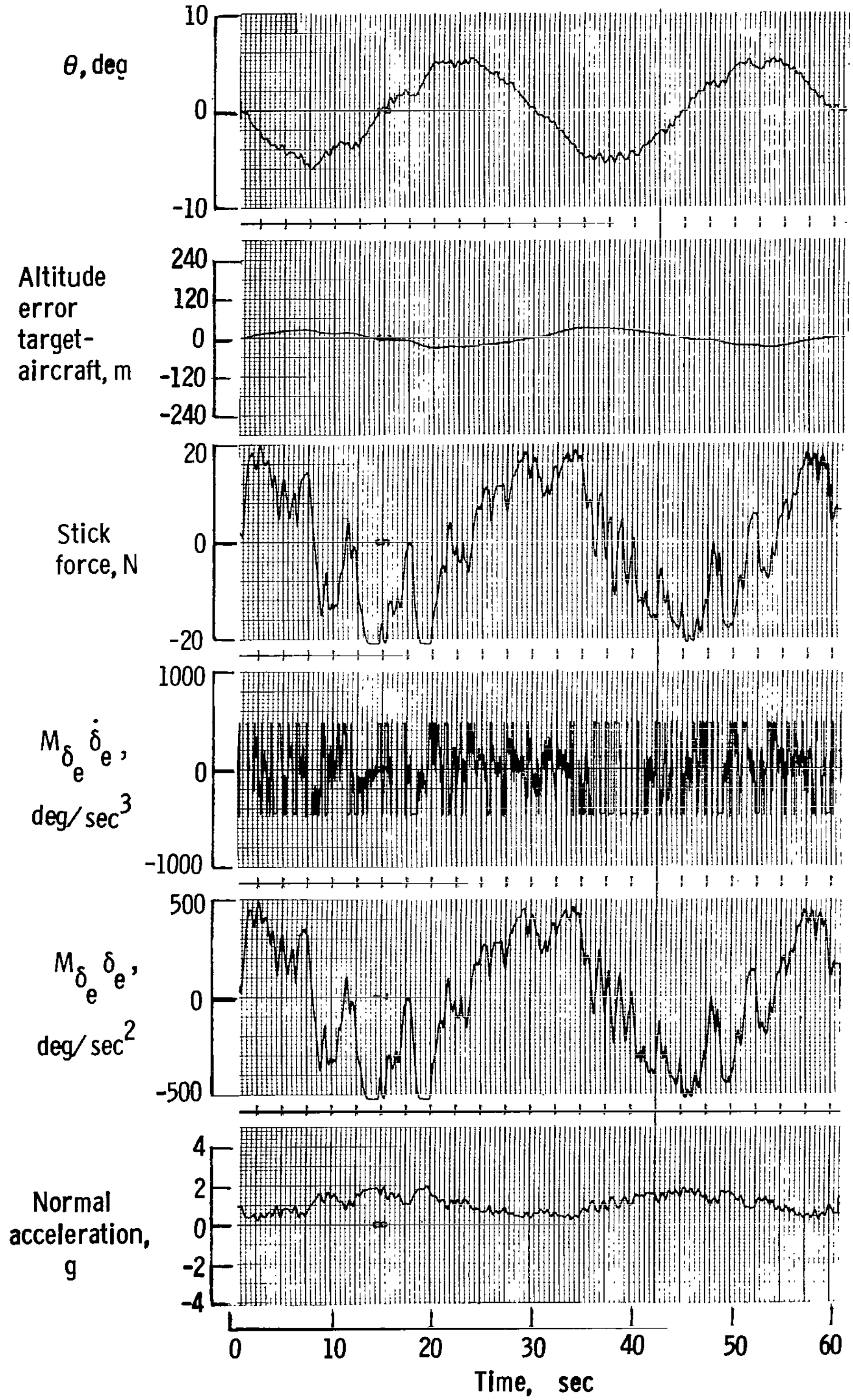
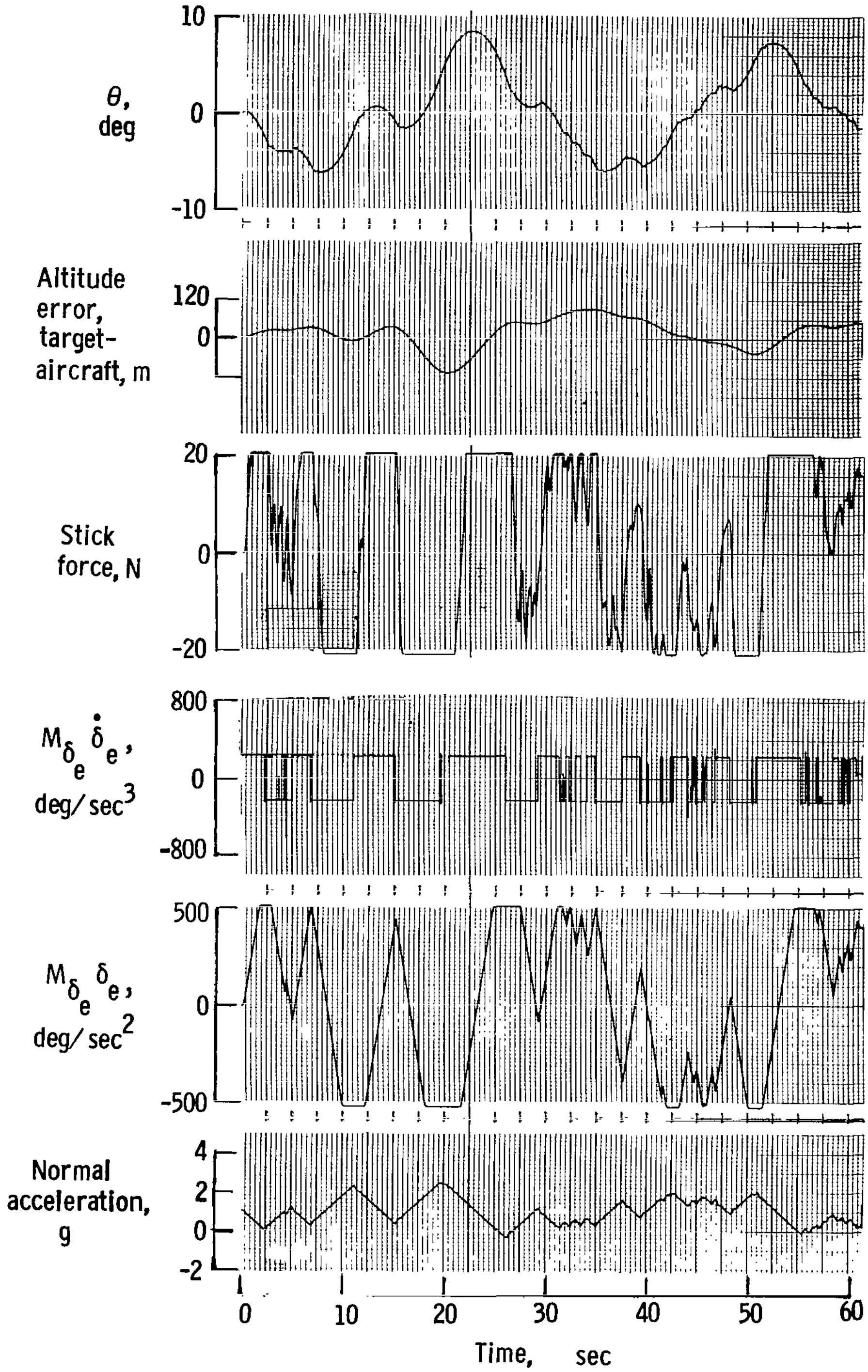


Figure 8.- Sinusoidal altitude command; low speed aircraft with nonlinear filter; $h_c = (120 \cos 0.21t) - 120$.



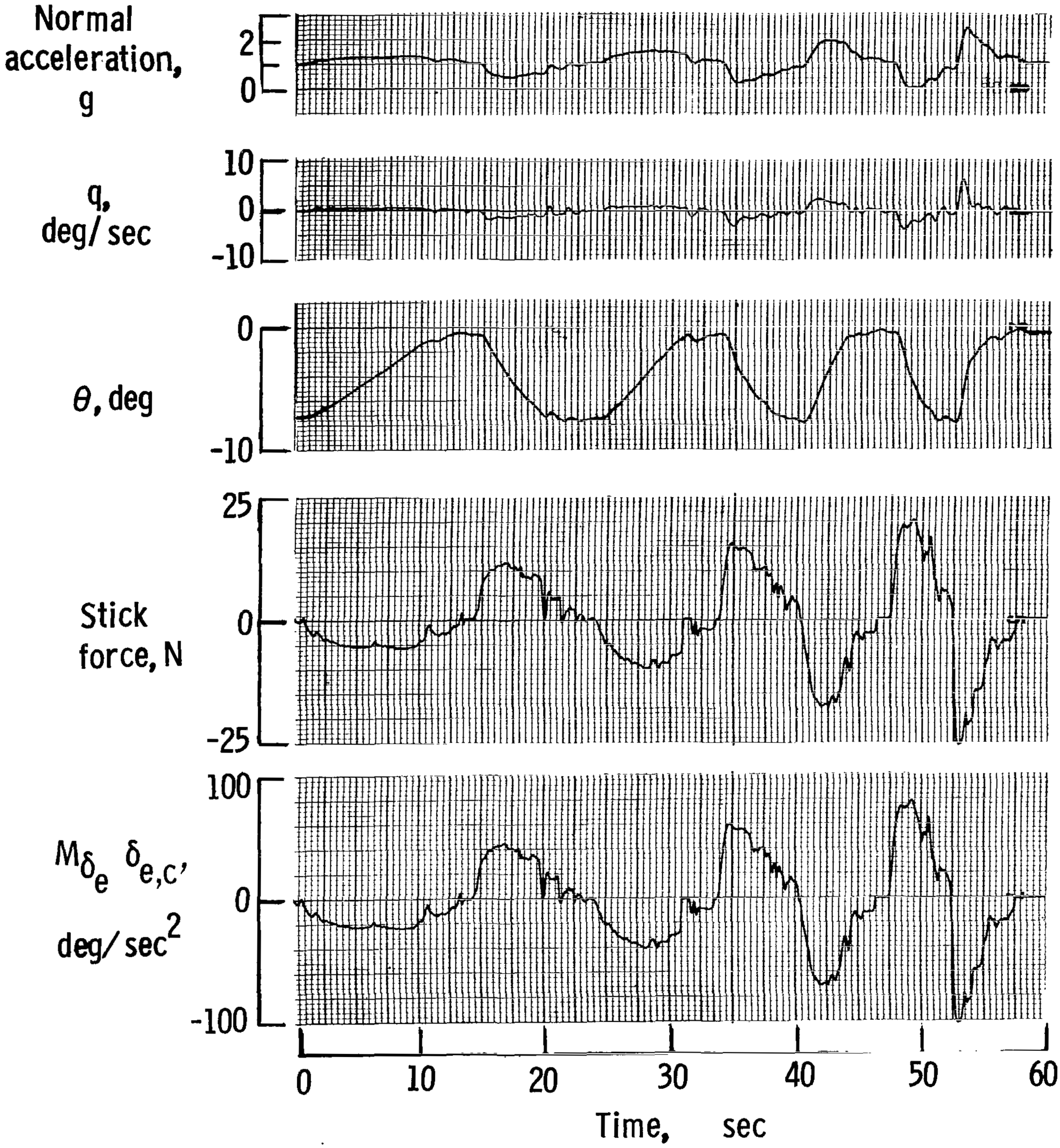
(a) Large rate limit in nonlinear filter.

Figure 9.- Sinusoidal altitude command; high-speed aircraft; $h_c = (120 \cos 0.21t) - 120$.



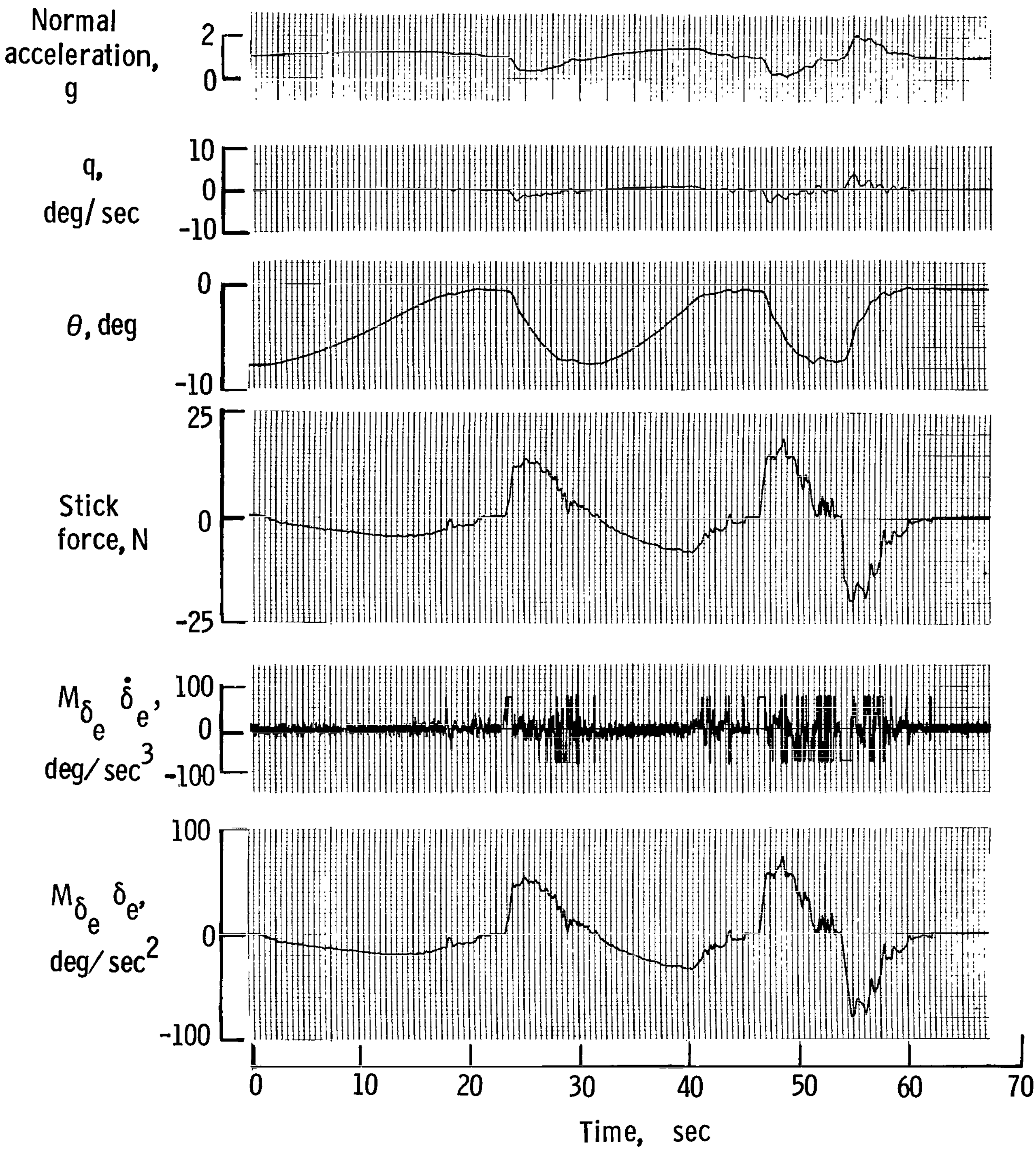
(b) Low rate limit in nonlinear filter.

Figure 9.- Concluded.



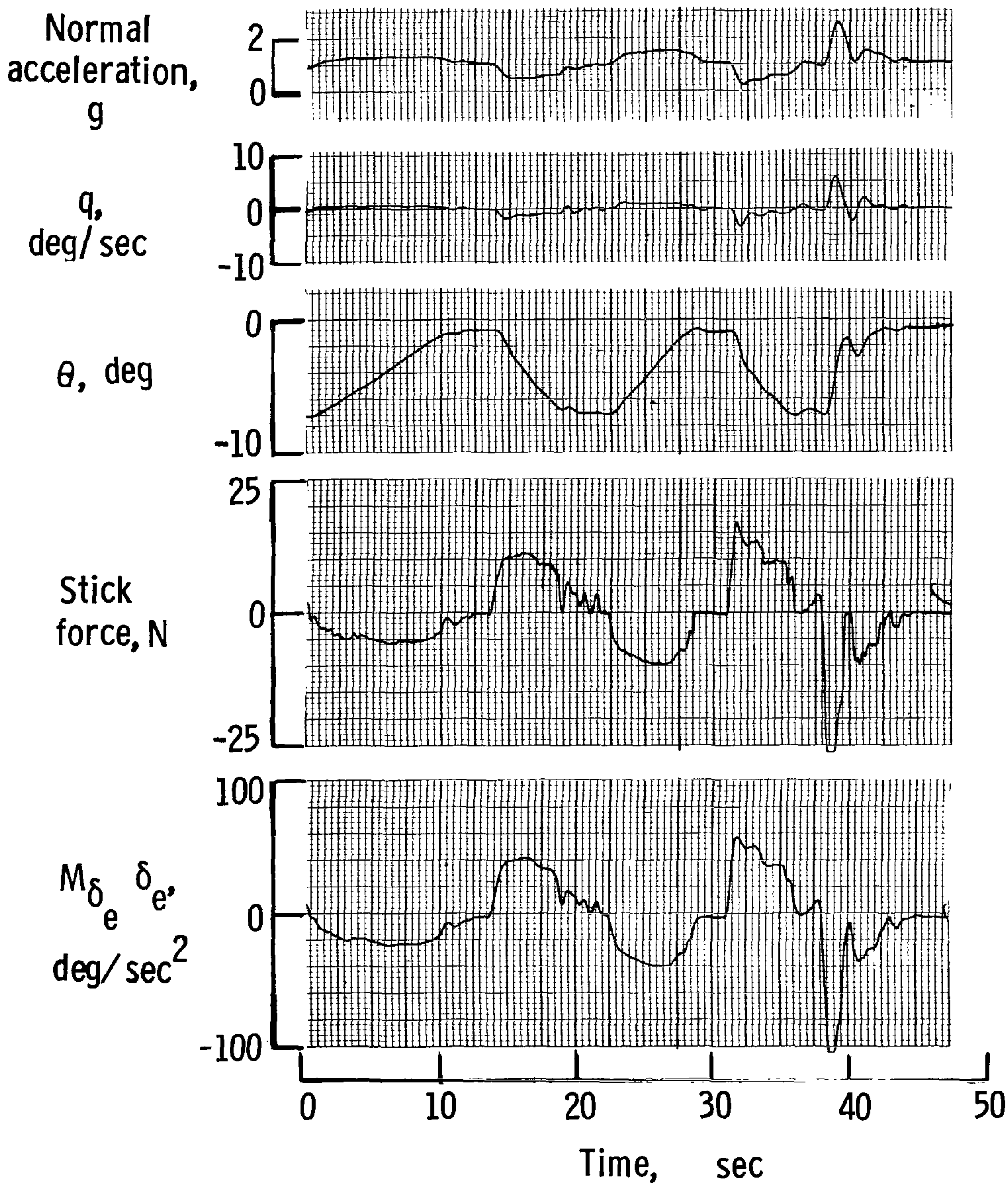
(a) No filter.

Figure 10.- Step pitch-angle change; pilot P; medium speed aircraft.



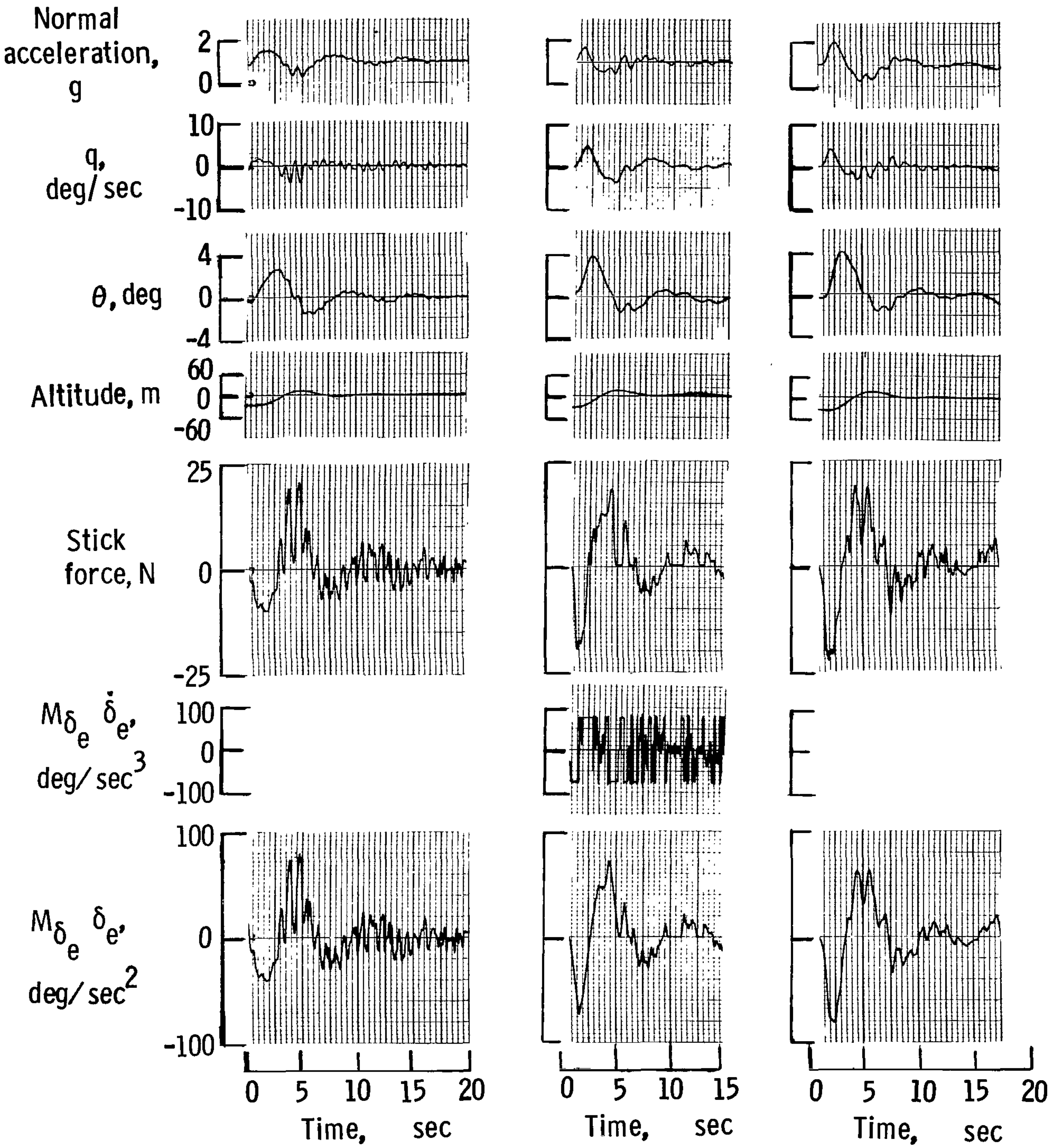
(b) Nonlinear filter.

Figure 10.- Continued.



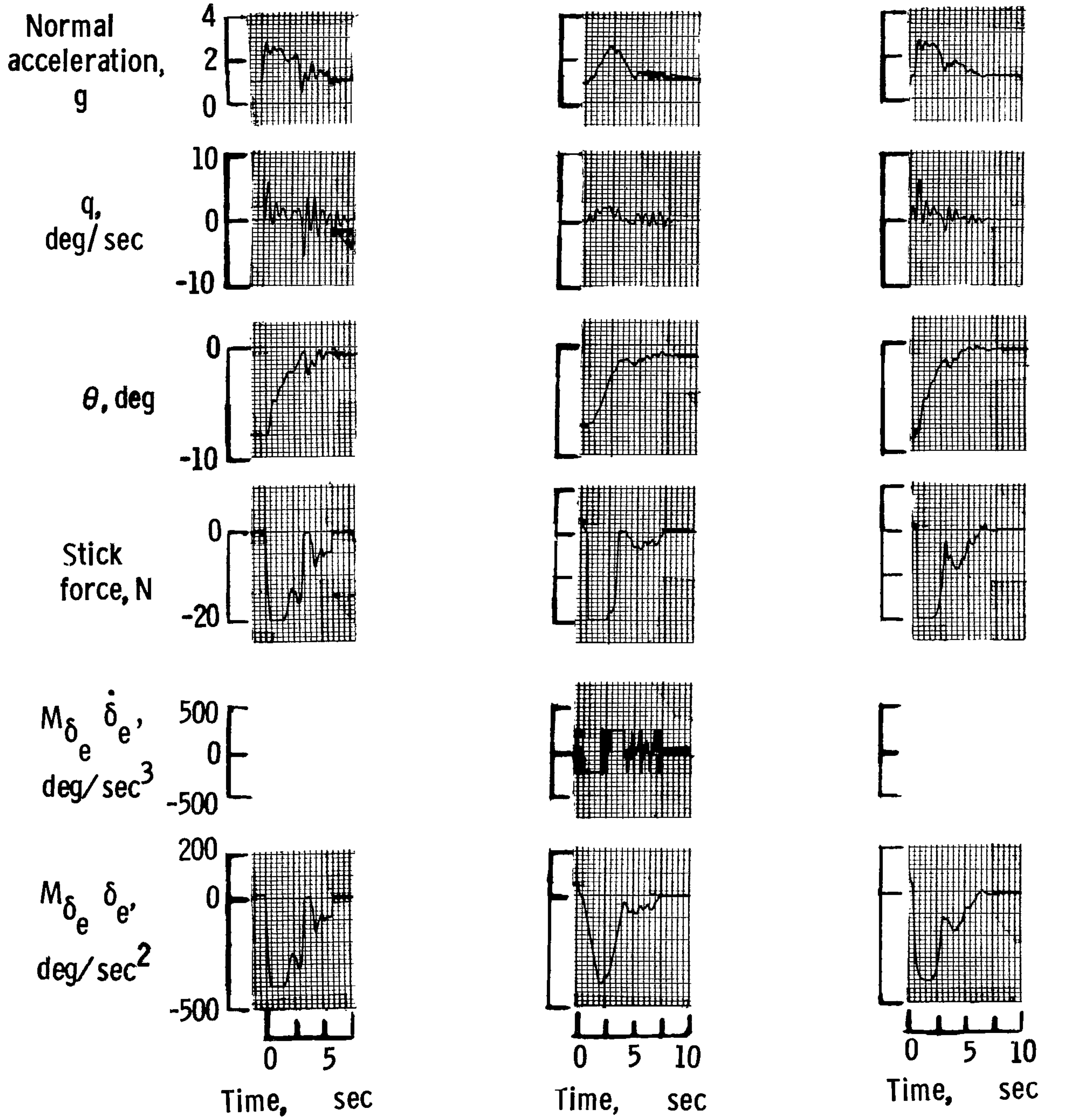
(c) Linear filter.

Figure 10.- Concluded.



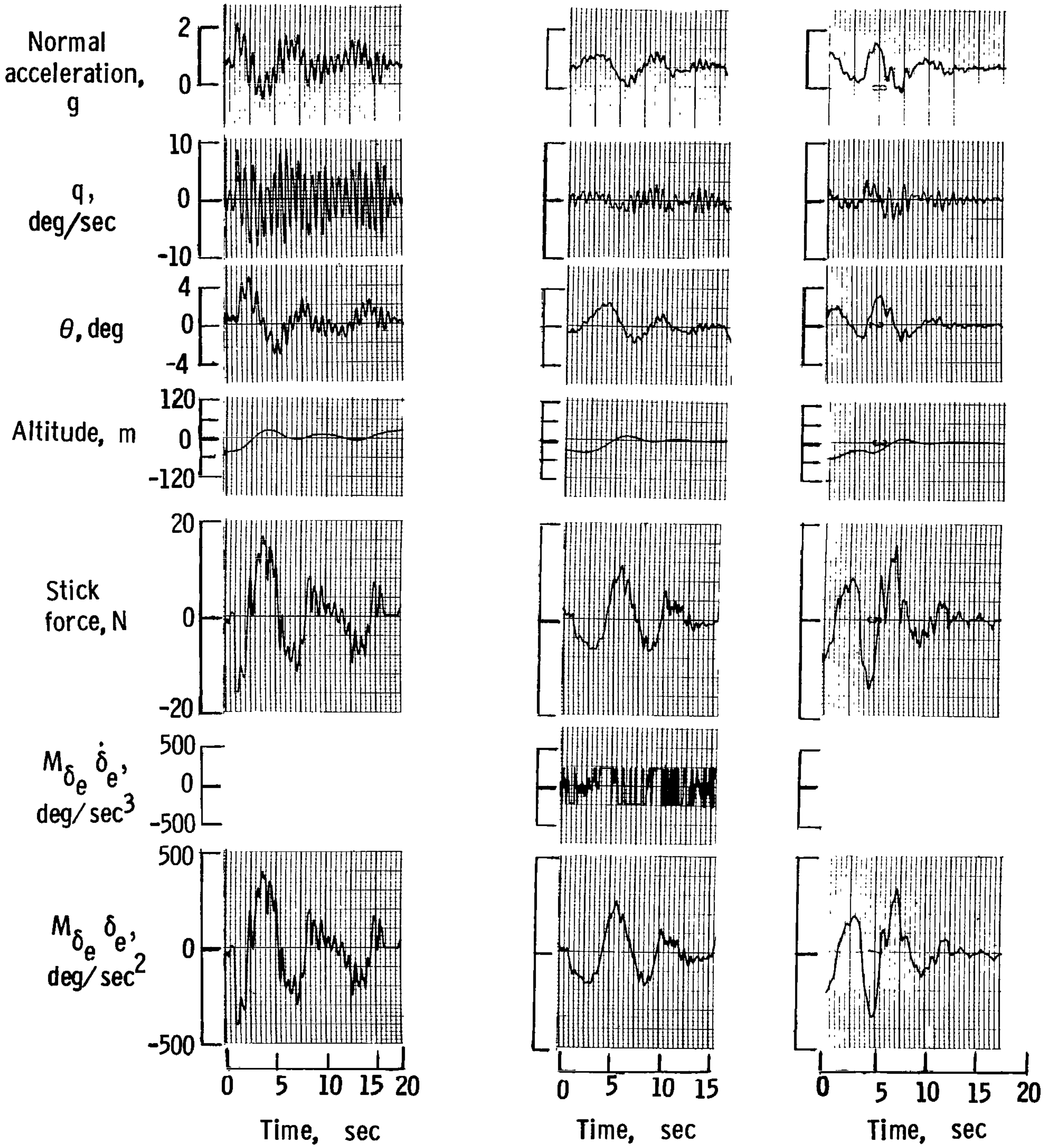
(a) No filter. (b) Nonlinear filter. (c) Linear filter.

Figure 11.- Step altitude change; pilot P; medium speed aircraft.



(a) No filter. (b) Nonlinear filter. (c) Linear filter.

Figure 12.- Step pitch-angle change; pilot P; high speed aircraft.



(a) No filter. (b) Nonlinear filter. (c) Linear filter.

Figure 13.- Step altitude change; pilot P; high speed aircraft.

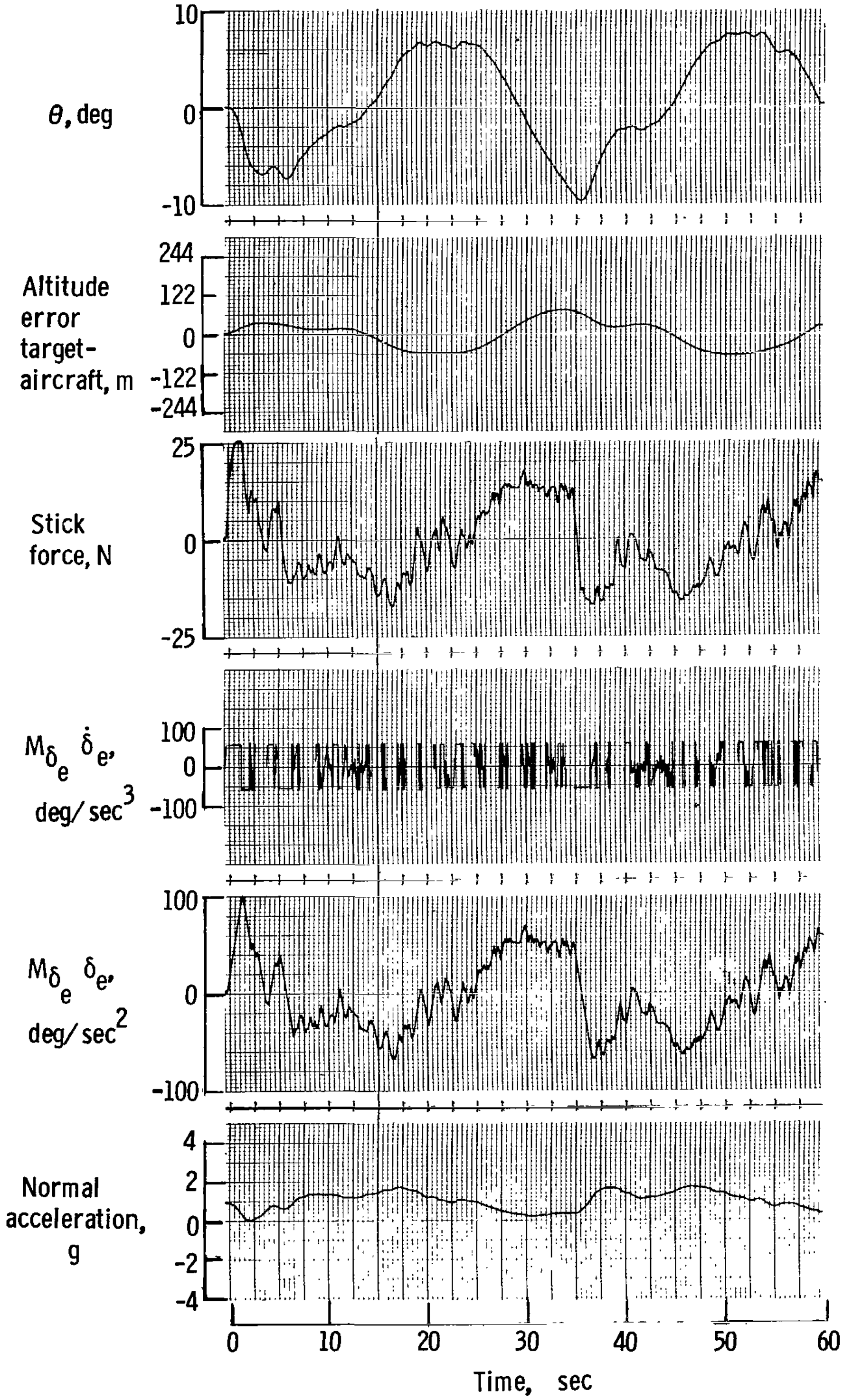


Figure 14.- Sinusoidal altitude command; low speed aircraft; nonlinear filter.

1. Report No. NASA TP-1147	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle STUDY OF THE USE OF A NONLINEAR, RATE-LIMITED FILTER ON PILOT CONTROL SIGNALS		5. Report Date April 1978
		6. Performing Organization Code
7. Author(s) James J. Adams		8. Performing Organization Report No. L-11762
		10. Work Unit No. 505-09-33-03
9. Performing Organization Name and Address NASA Langley Research Center Hampton, VA 23665		11. Contract or Grant No.
		13. Type of Report and Period Covered Technical Paper
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546		14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

The use of a nonlinear, rate-limited filter for rejecting the remnant (noise) in a pilot's control signal has been studied through both an analytical study using pilot models and a simulation study using experienced test pilots. The nonlinear filter was compared with a linear filter and with no filter for both attitude and altitude control tasks. The results show that the nonlinear filter does promote rapid, steady maneuvering better than either the linear filter or the no filter condition. In addition, if the rate limit in the nonlinear filter is set so that it is too restrictive, a pilot-induced unstable altitude oscillation can result.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Pilot-control prefilter Longitudinal control Pilot models	Unclassified - Unlimited Subject Category 08

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	38	$4.50

* For sale by the National Technical Information Service, Springfield, Virginia 22161

National Aeronautics and
Space Administration

Washington, D.C.
20546

Official Business
Penalty for Private Use, $300

THIRD-CLASS BULK RATE

Postage and Fees Paid
National Aeronautics and
Space Administration
NASA-451

U.S.MAIL

1 1 1U,A, 031378 S00903DS
DEPT OF THE AIR FORCE
AF WEAPONS LABORATORY
ATTN: TECHNICAL LIBRARY (SUL)
KIRTLAND AFB NM 87117

NASA

POSTMASTER: If Undeliverable (Section 158
Postal Manual) Do Not Return

S